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SOLUTION OF THE LONG ROD PENETRATION EQUATIONS

WILLIAM P. WALTERS
STEVEN B. SEGLETES

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U.S. ARMY LABORATORY COMMAND

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An exact solution is presented for the long rod penetration equations first formulated by Alekseevski in 1966 and independently by Tate in 1967. This analytical solution allows a faster and easier solution of the penetration equations, since stability considerations associated with any numerically integrated solutions are avoided. Additionally, an analytical solution provides greater insight into the penetration mechanism than a comparable numerically integrated solution.

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1. INTRODUCTION

The impact of a long, slender, eroding rod at high speed on a thick semi-infinite target was initially formulated by Alekseevski (1966) and Tate (1967, 1969). The governing equations, using the notation of Wright and Frank (1988), are:

$$\dot{L} = V - U, \tag{1}$$

$$L\,\dot{U} = -Y / \rho_r, \tag{2}$$

$$1/2\ \rho_r\,(U-V)^2 + Y = 1/2\ \rho_t\,V^2 + R, \tag{3}$$

and

$$\dot{P} = V. \tag{4}$$

where U is the speed of the rear of the penetrator, L is the instantaneous penetrator length, V is the penetration velocity, P is the depth of penetration, ρ_r is the penetrator density, Y is the penetrator yield stress, ρ_t is the target density, and R is the target resistance. In the equations, a dotted quantity represents the time derivative, d/dt.

Wright and Frank (1988) and Frank and Zook (1987) discuss these equations in detail, including the assumptions made in the derivation and approximate solutions. Our intent is to analyze the mathematical, not the physical, aspects of the equations (1) through (4). Basically, L, U, V, and P are the unknown, dependent variables, t (the time) is the independent variable, and ρ_r, Y, ρ_t, and R are known constants.

2. THE SOLUTION

An exact analytical solution of equations (1) - (4) for L, U, V, and P is now obtained. From equation (3)

$$v = \frac{u - \sqrt{\gamma u^2 + \Sigma(1-\gamma)}}{1-\gamma}, \tag{5}$$

where, if U_o is the initial (known) penetrator velocity, $v=V/U_o$, $u=U/U_o$, $\gamma=\rho_t/\rho_r$, and $\Sigma=2(R - Y)/(\rho_r\,U_o^2)$. The minus sign is chosen for the radical in the solution of the quadratic equation (3), to guarantee that V remains less than U, with both U and V real. Note that for the case R > Y ($\Sigma > 0$), the minimum admissable value of v is zero, corresponding to the moment that penetration ceases, which occurs at u= $\sqrt{\Sigma}$. For the case of R < Y ($\Sigma < 0$), the minimal admissable value of u is $\sqrt{-\Sigma/\gamma}$, in order to keep the root real in (5). In this case v = u, corresponding to the situation where rod erosion ceases and rigid body penetration commences.

Next, from equation (2), with $K = Y/(\rho_r U_o^2)$,

$$L\,\dot{u} = -K\,U_o. \tag{6}$$

Differentiation gives

$$L\,\ddot{u} + \dot{u}\,\dot{L} = 0,$$

and the solution for dL/dt, eliminating L, is

$$\dot{L} = \frac{K U_o \ddot{u}}{\dot{u}^2}.$$

Alternately, from equations (1) and (5),

$$\dot{L} = \frac{U_o}{(1-\gamma)}\left\{u\gamma - \sqrt{\gamma u^2 + \Sigma(1-\gamma)}\right\}.$$

Combining these two expressions to eliminate dL/dt gives

$$\frac{\ddot{u}}{\dot{u}} = \frac{\gamma u \dot{u}}{K(1-\gamma)} - \frac{\dot{u}\sqrt{\gamma u^2 + \Sigma(1-\gamma)}}{K(1-\gamma)}. \tag{7}$$

Straightforward integration yields

$$ln\left(\dot{u}\left\{u\sqrt{\gamma} + \sqrt{\gamma u^2 + \Sigma(1-\gamma)}\right\}^{\Sigma/2K\sqrt{\gamma}}\right)$$

$$= \frac{\gamma u^2}{2K(1-\gamma)} - \frac{u}{2K(1-\gamma)}\sqrt{\gamma u^2 + \Sigma(1-\gamma)} + G, \tag{8}$$

where G is a constant of integration which results from evaluation of the integral at the onset of penetration, when $u = 1$ and $\dot{u} = \dot{u}_o$. Note that $\dot{u}_o$, which equals $(1/U_o)dU/dt|_o$, and has dimensions of [1/t], can be evaluated from equation (6) as $\dot{u}_o = -K\, U_o/L_o$. The constant G may be expressed as

$$G = N + ln M + ln \dot{u}_o ,$$

where

$$M = \left\{ \sqrt{\gamma} + \sqrt{\gamma + \Sigma(1-\gamma)} \right\}^{\Sigma/(2K\sqrt{\gamma})} ;$$

$$N = \frac{\sqrt{\gamma + \Sigma(1-\gamma)}}{2K(1-\gamma)} - \frac{\gamma}{2K(1-\gamma)} .$$

By substituting the constant A for the exponent, $\Sigma/(2K\sqrt{\gamma})$, equation (8) may be expressed in the following form:

$$ln\left[\frac{\dot{u}}{\dot{u}_o}\ \frac{\left\{u\sqrt{\gamma} + \sqrt{\gamma u^2 + \Sigma(1-\gamma)}\right\}^A}{M}\right]$$

$$= \frac{\gamma u^2}{2K(1-\gamma)} - \frac{u}{2K(1-\gamma)}\sqrt{\gamma u^2 + \Sigma(1-\gamma)} + N . \qquad (9)$$

By introducing the following transformation variable z,

$$\sqrt{z} = u\sqrt{\gamma} + \sqrt{\gamma u^2 + \Sigma(1-\gamma)} . \qquad (10)$$

equation (9), under the transformation (10), yields

$$\left(z^{(A-1)/2} + \Sigma(1-\gamma)\, z^{(A-3)/2}\right)\exp(Bz - C/z)\, dz = F\, dt, \qquad (11)$$

where

$$B = \frac{1}{8K\sqrt{\gamma}\,(\sqrt{\gamma}+1)},$$

$$C = \frac{\Sigma^2(1-\gamma)(\sqrt{\gamma}+1)}{8K\sqrt{\gamma}},$$

and

$$F = 4\dot{u}_o M\sqrt{\gamma}\,\exp\left[N - \frac{\Sigma}{4K}\right].$$

The use of the transformation (10) produces a differential equation (11), in which the variables, z and t, are now separable. If integrated from time 0 to some finite time t in the penetration process, the limits on z will vary from its initial value, when u = 1, of

$$z_o = \left(\sqrt{\gamma} + \sqrt{\gamma + \Sigma(1-\gamma)}\right)^2,$$

to some intermediate value z. For the case of R > Y, the terminal value of time at which the governing equations are applicable occurs when penetration ceases at v = 0, in which case $u = \sqrt{\Sigma}$ and the terminal value of z, expressed as z_z, is given by

$$z_z = \Sigma\left(\sqrt{\gamma}+1\right)^2.$$

For the case of R < Y, the long rod penetration equations are only valid (without modification) to the time at which the penetrator begins rigid body penetration, in which case $u = v = \sqrt{-\Sigma/\gamma}$, and the terminal value of z becomes

$$z_z = (-\Sigma)\left(\sqrt{\gamma}+1\right)^2.$$

Note that z is always positive, since Σ is positive for R > Y, and (-Σ) is also positive for R < Y.

Equation (11) may be further simplified by letting

$$\phi = z^{(A+1)/2}$$

in the first integral over z and

$$\theta = z^{(A-1)/2}$$

in the second integral over z. Under these transformations, and letting $E_1=2/(A+1)$ and $E_2=2/(A-1)$, the integration of equation (11) reduces to

$$E_1 \int_{\phi_o}^{\phi} \exp(B\phi^{E_1} - C\phi^{-E_1})\, d\phi$$

$$+ \Sigma(1-\gamma) E_2 \int_{\theta_o}^{\theta} \exp(B\theta^{E_2} - C\theta^{-E_2})\, d\theta = F \int_{o}^{t} dt\,. \tag{12}$$

The solution is now reduced to a straightforward integration, though it requires evaluation of an exponential integral which, in theory, is a tabulated function of five input parameters. Defining the function, W, as

$$W(B,C,E,y_1,y_2) = E \int_{y_1}^{y_2} \exp\left(By^{E} - Cy^{-E}\right) dy\,, \tag{13}$$

the solution for t becomes simply

$$t = \left[W(B,C,E_1,\phi_o,\phi) + \Sigma(1-\gamma)W(B,C,E_2,\theta_o,\theta)\right]/F\,.$$

In practice, our function is evaluated by expanding the exponential function in equation (13) in a power series and integrating term by term, to the desired degree of precision. The result is z as an implicit function of the time variable, t.

The number of power series terms required for convergence of our W function varies a great deal with the input conditions to the problem. In particular, the evaluation of our W function by way of power series can be exacerbated for problems where the penetrator velocity overwhelms the strengths of the rod and target materials. Fortunately, problems in this velocity range are generally beyond the range of interest, for typical long rod penetrator impacts.

Because B is always positive, the first part of the exponential term grows with z. As B is made parabolically larger by increasing the penetrator striking velocity U_o, more terms are required to make the power series converge. Because it is a binomial that needs to be exponentiated, use of n terms in the exponential expansion requires that n(n-1)/2 monomials be evaluated. Similarly, the coefficient for each of the n highest order monomials requires (2n) operations to evaluate. Thus, the computational effort required to evaluate our W function varies greatly with initial conditions to the problem.

Typical penetration problems involving significant, but not total, penetrator erosion require that 10 to 20 exponential terms be evaluated in order to keep the relative error of the time variable in the fifth decimal place. Such calculations require mere seconds of computation on a PC. As hypervelocity conditions are approached, the number of exponential terms required for the same convergence epsilon may exceed 200 (recall that 200 exponential terms implies 200 x 199/2 = 19,900 monomials), requiring several minutes on a PC. Fortunately, this solution technique need not be pursued for problems in hypervelocity since, under these conditions, the long rod penetration equations approach the standard Bernoulli flow conditions, which may be readily solved by hand.

Having t as a function of z, the normalized rod speed, u, follows from equation (10) as

$$u = \frac{z - \Sigma(1-\gamma)}{2\sqrt{\gamma}\,\sqrt{z}} . \tag{14}$$

Equation (5) may then be employed to obtain the normalized penetration velocity, v. The rate of rod erosion comes from equation (1), in the form

$$\dot{L} = U_o(v - u) .$$

The penetrator length, L, may be obtained in the following fashion. From equations (1) and (2), one obtains

$$\frac{\dot{L}}{L} = \frac{(u-v)\dot{u}}{K}.$$

Substituting for v and u give the following:

$$-\frac{\dot{L}}{L} = \frac{\gamma u \dot{u}}{K(1-\gamma)} - \frac{\dot{u}\sqrt{\gamma u^2 + \Sigma(1-\gamma)}}{K(1-\gamma)}.$$

Note the identical form of this relation and equation (7). As a result, the solution looks nearly identical to equation (9), including the definition of constants A, M, and N:

$$-ln(L/L_o) = -ln\left[\frac{\left\{u\sqrt{\gamma} + \sqrt{\gamma u^2 + \Sigma(1-\gamma)}\right\}^A}{M}\right]$$

$$+ \frac{\gamma u^2}{2K(1-\gamma)} - \frac{u}{2K(1-\gamma)}\sqrt{\gamma u^2 + \Sigma(1-\gamma)} + N.$$

Finally, the penetration, P, is obtained as follows:

$$P = \int_o^t V\,dt = U_o \int_o^t v\,(dt/dz)\,dz. \qquad (15)$$

The quantity dt/dz has been previously obtained in equation (11) as

$$\frac{dt}{dz} = \frac{\left(z^{(A-1)/2} + \Sigma(1-\gamma)z^{(A-3)/2}\right)\exp(Bz - C/z)}{F}.$$

We may express v in terms of z, using equations (5) and (14), as follows:

$$v = \frac{1}{2\sqrt{\gamma}}\left[\frac{\left(1-\sqrt{\gamma}\right)\sqrt{z}}{(1-\gamma)} - \frac{\Sigma\left(1+\sqrt{\gamma}\right)}{\sqrt{z}}\right].$$

These substitutions into (15) produce the following expression for P:

$$P = U_o/F \int_{z_o}^{z} \left[\frac{1-\sqrt{\gamma}}{2\sqrt{\gamma}(1-\gamma)} z^{A/2} - \Sigma z^{(A-2)/2} - \frac{\Sigma^2(1+\sqrt{\gamma})(1-\gamma)}{2\sqrt{\gamma}} z^{(A-4)/2} \right] exp(Bz - C/z)\, dz\,.$$

This equation is similar to Equation 11, in that it may be solved directly for penetration P, in this case, in terms of serveral W functions.

The results of the present solution have been compared with the original results presented by Tate (1967), in which he numerically integrated the penetration-time history of a duralumin rod strinking a polythene target, for two different target resistance values. The curves resulting from the present analytical solution achieve a direct overlay to Tate's numerically integrated solution.

Finally, the source code listing for the penetration equations considered in this report is given in the Appendix.

3. SPECIAL CASE SOLUTIONS

Two special cases are considered:

A) $\rho_r = \rho_t = \rho$, and $Y = R = \sigma$

B) $\rho_r = \rho_t = \rho$.

SPECIAL CASE A: $\rho_r = \rho_t = \rho$, and $Y = R = \sigma$.

Equation (3) reduces to

$$(u-v)^2 = v^2\,,$$

with the non-trivial solution $u = 2v$. Differentiating equation (2), and combining the result with equation (1), as before, yields:

$$\frac{\ddot{u}}{\dot{u}} = \frac{2u\dot{u}}{-H},$$

where

$$H = \frac{4\sigma}{\rho U_o^2}.$$

Integrating this equation and evaluating the constant of integration at time equal 0, where $\dot{u} = \dot{u}_o$, which has the value $\dot{u}_o = (-H\ U_o)/(4L_o)$, results in

$$\dot{u} = \dot{u}_o \exp\left[\frac{1-u^2}{H}\right].$$

Integrating again for u gives the result in terms of our W function as

$$t = \frac{\exp(-1/H)}{2\dot{u}_o} W(1/H, 0, 2, 1, u),$$

which gives u implicitly as a function of time, t. As mentioned above, u = 2v applies, and thus determines penetration rate v. Also,

$$\dot{L} = \frac{-U_o u}{2},$$

and

$$L = \frac{-H U_o}{4\dot{u}}$$

follow directly. To determine penetration P, employ the tactic of equation (15), namely

$$P = \int_o^t V dt = U_o \int_1^u v\,(dt/du)\,du.$$

Penetration rate, v, is known directly in terms of u, equal to (u/2), and dt/du is simply $1/\dot{u}$, given by

$$\frac{dt}{du} = \frac{1}{\dot{u}_o} \exp\left[\frac{u^2-1}{H}\right].$$

Thus, making use of the term $\dot{u}_o = (-H\ U_o)/(4L_o)$, the penetration may be computed as

$$P = L_o\left\{1 - \exp\left[\frac{u^2-1}{H}\right]\right\}.$$

SPECIAL CASE B: $\rho_r = \rho_t = \rho$.

Equation (3), which is no longer quadratic, as in the general case, yields

$$v = \frac{1}{2}(u - \Sigma/u).$$

Differentiating equation (2), and combining the result with equation (1), and the expression for v above, yields

$$K\frac{\ddot{u}}{\dot{u}} = -\frac{u\dot{u}}{2} - \frac{\Sigma\dot{u}}{2u}.$$

Direct integration results in

$$\dot{u} = \dot{u}_o u^{-A} \exp\left[\frac{1-u^2}{4K}\right],$$

where the exponent A has the value, analogous to the general case derivation, of $\Sigma/(2K)$. The variables are separable, and

$$\int_1^u u^A \exp\left[\frac{u^2-1}{4K}\right] du = \dot{u}_o \int_0^t dt.$$

This integral is evaluated using the same procedure used for equation (11). By letting

$$\phi = u^{(A+1)},$$

the integral reduces to

$$\frac{1}{(A+1)} \int_1^\phi \exp\left[\frac{\phi^{2/(A+1)}-1}{4K}\right] d\phi = \dot{u}_o t,$$

the evaluation of which may be expressed in terms of our W function as

$$t = \frac{1}{2\dot{u}_o} \exp\left[\frac{-1}{4K}\right] W\left((4K)^{-1}, 0, 2/(A+1), 1, u^{(A+1)}\right).$$

The evaluation of the remaining variables v, dL/dt, L, P follows an approach analogous to the general case.

4. EXTENDING THE SOLUTION

The long rod equations (1)-(4) do not hold for the complete penetration process. In particular, if R > Y, the equations are only valid until penetration ceases (V = 0), even though rod erosion is still occuring (U > 0). On the other hand, for R < Y, the equations (1)-(4) are only valid until rod erosion ceases (V = U), even though rigid body penetration proceeds (V = U > 0).

When either of these limiting conditions occurs, the equations (1)-(4) can no longer remain valid without some sort of alteration, in order to prevent the situations V < 0 or U < V. To illustrate this point, consider adding the constraint of setting V to zero, when R > Y, to obtain

$$\dot{L} = -U,$$
$$L\,\dot{U} = -Y/\rho_r,$$
$$1/2\,\rho_r\,U^2 + Y = R, \text{ and}$$
$$\dot{P} = 0$$

or setting the term V = U, if R < Y, to obtain

$$\dot{L} = 0,$$
$$L\,\dot{U} = -Y/\rho_r,$$
$$Y = 1/2\,\rho_t\,U^2 + R, \text{ and}$$
$$\dot{P} = U.$$

Since V has been eliminated as a dependent variable, leaving just U, L, and P as unknowns, the stipulation of four governing equations overconstrains the problem. If no other conditions are changed, then the third equation of each set (modified Bernoulli equation) amounts to setting the rod and/or penetration velocity to a constant, which is clearly incorrect. One possibility involves the elimination of the modified Bernoulli equation directly, on the assumption that the U, V relationship it defines is replaced by the rigidity constraints: V = 0 if R > Y, or V = U if R < Y.

Another possible remedy to this problem, and the alternative being proposed, involves introducing an additional unknown, in the form of a material resistance R or Y, to vary from its initial value of R_o or Y_o, respectively.

The rationale for this step is given now. The term R and Y represent deviatoric stress resistances built up in the target and penetrator, respectively, which act as part of a force balance in the modified Bernoulli in equation (3). For the case of $R_o > Y_o$, the target penetration ceases at some point while rod erosion continues. After this point of target rigidity, the target is elastic. Thus, the resistive stress offered by the target (R) will decrease from its full plastic value (R_o), to an elastic value exactly equal to the penetrator yield strength (Y_o), just as the penetrator velocity becomes identically zero. For the alternative case of $R_o < Y_o$, the rod erosion ceases at some point while rigid body penetration continues. After this point of penetrator rigidity, the penetrator is elastic. Thus, the resistive stress offered by the penetrator (Y) will decrease from its full plastic value (Y_o) to an elastic value exactly equal to the target yield strength (R_o), just as the penetration velocity becomes identically zero.

These equation sets may then be solved to determine the residual rod erosion (if R > Y) or residual penetration (if R < Y), and the associated event duration, using the appropriate initial conditions, which resulted from the terminal conditions associated with the previous solution of Equations (1)-(4). The boundary conditions are given below, with subscript r referring to conditions at the onset of rigidity (target rigidity if R > Y or rod rigidity if R < Y), and subscript x referring to conditions of U = V = 0.

<u>Target Rigid: R > Y (V = 0)</u>

Initial Conditions	Final Conditions
$U = U_r$	$U = 0$
$L = L_r$	$L = L_x$
$P = P_r$	$P = P_r$
$t = t_r$	$t = t_x$
$R = R_o$	$R = Y_o$
$Y = Y_o$	$Y = Y_o$

<u>Penetrator Rigid: R < Y (V = U)</u>

Initial Conditions	Final Conditions
$U = U_r$	$U = 0$
$L = L_r$	$L = L_r$
$P = P_r$	$P = P_x$
$t = t_r$	$t = t_x$
$R = R_o$	$R = R_o$
$Y = Y_o$	$Y = R_o$

For the case of R > Y, the solution is expressable in terms of the W function described in the original section of this report. Non-dimensional terms (e.g., K, γ, u) defined earlier are also employed, to give the solution as:

$$L_s = L_r \exp -\left[(R_o - Y_o)/Y_o\right],$$

$$t_s - t_r = \frac{L_s}{2K_o U_o} W\left[(2K_o)^{-1}, 0, 2, 0, U_r\right].$$

The term K_o refers to the value of K at its original constant value, when $R = R_o$ and $Y = Y_o$. For the case of $R < Y$, the solution is determined as:

$$P_s - P_r = (L_r/\gamma)\ln(Y_o/R_o)$$

$$t_s - t_r = \frac{L_r}{U_o}\left[\frac{2Y_o}{\gamma K_o R_o}\right]^{1/2} \tan^{-1}\left[\frac{U_r}{\left[\frac{2K_o R_o}{\gamma Y_o}\right]^{1/2}}\right].$$

The results are again compared to Tate (1967), in which the penetration of a duralumin rod into polythene was modeled for two values of target resistance—14 and 27 tons per square inch. Tate pointed out that these two values produced curves which straddled the actual penetration-time data. In Tate's formulation, the equations are only valid to the point where rigid-body penetration commences which, for the case in question, accounts for only 80% of the total penetration. Using the current formulation, for computing the residual, rigid-body penetration of the duralumin rod, the experimental value of penetration may be used to compute the exact value of target resistance required. For this case, using the units of Tate, a target resistance of 20 tons per square inch was computed as necessary to produce the experimentally observed residual penetration, measured from Tate's graph as 6.15 inches. The penetration process is predicted to stop at 242 μsec after impact, thus implying that rigid body penetration takes 42% of the total time of penetration.

5. CONCLUSIONS

An analytical solution to the long rod penetration equations for long rod penetration is offered. The general case is solved as well as two special cases in which some of the target and penetrator parameters (e.g., density and/or strength) are equal. This analytical solution allows a faster and easier solution of the penetration equations, since stability considerations associated with any numerically integrated solution are avoided.

Additionally, the proposed modification to the original model permits the computation of residual rod erosion and residual penetration, which offer additional information about the penetration process, not available in the original penetration equations formulated by Tate.

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6. REFERENCES

Alekseevski, V. P. "Penetration of a Rod into a Target at High Velocity." Combustion, Explosion, and Shock Waves, vol. 2, pp. 99-106, 1966.

Frank, K., and J. Zook. "Energy Efficient Penetration and Perforation of Targets in the Hypervelocity Regime." International Journal of Impact Engineering, vol. 5, pp. 277-284, 1987.

Tate, A. "A Theory for the Deceleration of Long Rods After Impact." Journal of the Mechanics and Physics of Solids, vol. 15, pp. 387-399, 1967.

Tate, A. "Further Results in the Theory of Long Rod Penetration." Journal of the Mechanics and Physics of Solids, vol 17, pp. 141-150, 1969.

Wright, T.W., and K. Frank. "Approaches to Penetration Problems." BRL-TR-2957, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, December 1988.

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APPENDIX:
FORTRAN SOURCE CODE LISTING

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```
      program tate
C
C     CODED BY Steven B. Segletes, AUGUST-SEPTEMBER, 1990.
C
C     THIS PROGRAM IMPLEMENTS WALTERS' AND SEGLETES' ANALYTICAL
C     SOLUTION OF THE TATE EQUATIONS:
C
C     dL/dt = V - U
C
C     L dU/dt = -Y/@r
C                    2                2
C     1/2 @r (U-V)   + Y = 1/2 @t V   + R
C
C     dP/dt = V
C
C     WHERE:
C     V  - ROD VELOCITY
C     U  - PENETRATION VELOCITY
C     Y  - TATE ROD STRENGTH PARAMETER
C     @r - ROD DENSITY
C     R  - TATE TARGET STRENGTH PARAMETER
C     @t - TARGET DENSITY
C     L  - ROD LENGTH
C     P  - PENETRATION
C     t  - TIME
C
C     =====VARIABLES=====
C
C     *****DIMENSIONAL TATE CONSTANTS*****
C
C     R    - TARGET RESISTANCE
C     Y    - ROD RESISTANCE
C     RHOT - TARGET DENSITY
C     RHOR - ROD DENSITY
C     Lo   - INITIAL ROD LENGTH
C     Uo   - INITIAL ROD VELOCITY
C
C     *****NON-DIMENSIONAL CONSTANTS*****
C
C     GAMMA
C     SIGMA
C     K
C     A
C     BIGB
C     C
C     M
C     N
C     F     - A CONGLOMERATION OF CONSTANTS, APPEARING OCCASIONALLY
C     Z0    - INITIAL VALUE OF Z (WHEN U = Uo)
C     ZX    - TERMINAL VALUE OF Z (WHEN V = 0)
C
C     *****OUTPUT VARIABLES*****
C
C     Z()   - ARRAY OF TRANSFORMATION VARIABLES, AT WHICH SOLUTION
C             IS TO BE EVALUATED
C     T()   - ARRAY OF TIMES, CORRESPONDING TO THE TRANSFORMATION
C             Z VARIABLES
C     U     - ROD VELOCITY
C     V     - PENETRATION VELOCITY
C     Ldot  - RATE OF ROD EROSION
C     L     - LENGTH OF UNERODED ROD
C     P()   - ARRAY OF PENETRATION VALUES CORRESPONDING TO THE
C             TRANSFORMATION Z VARIABLES
```

```
C     XTAIL - COORDINATE OF THE REAR END OF THE ROD
C
C     *****BOOKKEEPING VARIABLES
C
C     MXPS     - MAXIMUM NUMBER OF TIMES, LESS 1, AT WHICH SOLUTION MAY
C                BE EVALUATED
C     EPSILON  - RELATIVE ERROR TO BE USED IN TESTING FOR SERIES
C                CONVERGENCE OF THE W INTEGRAL
C     NTERMS   - NUMBER OF TERMS REQUIRED TO CONVERGE THE W
C                INTEGRAL
C     NTAVG    - AVERAGE NUMBER OF TERMS REQUIRED TO CONVERGE THE W
C                INTEGRAL, OVER THE 5 INTEGRAL EVALUATIONS REQUIRED.
C     NPASS    - NUMBER OF TIMES AT WHICH THE SOLUTION IS EVALUATED
C                (MAY NOT EXCEED MXPS+1)
C     I        - A LOOP INDEX
C     ANSWER   - A ONE CHARACTER RESPONSE TO THE QUERY FOR SCREEN OR FILE
C                OUTPUT
C
C     *****OTHER VARIABLES*****
C
C     ZZ     - SCALAR VERSION OF Z, CORRESPONDING TO THE SINGLE Z ELEMENT
C              OF CURRENT INTEREST
C     DT     - USER SPECIFIED DELTA TIME, AT WHICH HE HOPES TO HAVE SOLUTIONS
C              EVALUATED (NOTE THAT ACTUAL SOLUTION INTERVAL IS ONLY
C              APPROXIMATELY THIS VALUE)
C     DZ     - INCREMENT IN Z CORRESPONDING ROUGHLY TO THE TIME INCREMENT DT
C     Ur     - NON-DIMENSIONAL ROD VELOCITY (U/Uo)
C     Vr     - NON-DIMENSIONAL PENETRATION VELOCITY (V/Vo)
C
C     D          - AN EXPONENT ON Z, APPEARING IN THE W INTEGRAL
C     TERMx()    - WHERE x IS 1, 2 OR 3... ARRAYS CONTAINING EVALUATIONS OF
C                  THE W INTEGRAL, EACH ARRAY ELEMENT WITH DIFFERENT
C                  LIMITS OF INTEGRATION, AS GIVEN BY THE Z ARRAY
C     ROOTGAMMA  - SQUARE ROOT OF GAMMA, WHICH OCCURS FREQUENTLY
C     ROOTTERM   - A SQUARE ROOT TERM, APPEARING OCCASIONALLY
C     ROOTZ      - SQUARE ROOT OF Z, APPEARING OCCASIONALLY
C
      integer MXPS
      PARAMETER (MXPS=200)
      double precision gamma, sigma, K, A, BigB, c, F, z0, zx, M, N
      common /wconstants/ gamma, sigma, K, A, BigB, c, F, z0, zx, M, N
      double precision R, Y, rhot, rhor, Lo, Uo
      common /tconstants/ R, Y, rhot, rhor, Lo, Uo
      double precision rootgamma, z(0:MXPS), zz, D, term1(0:MXPS),
     &                 term2(0:MXPS), term3(0:MXPS), EPSILON, dt,
     &                 dz, t(0:MXPS), U, V, Ldot, L, P(0:MXPS), xtail,
     &                 rootterm, rootz, Ur, Vr, C1, E
      integer nterms, ntavg, npass, i
      character*1 answer
      PARAMETER (EPSILON=1.0D-3)
C
    1 format (' Enter approximate dt: ')
    2 format (1x,'   z         t          U           V          ',
     &        'dL/dt       L           P          Xtail')
    3 format (f6.2,1p,e9.2,2e10.3,e10.2,2e10.3,e11.3)
    4 format ('xxxxx','    8',/,
     &        'R,Y,@t,@r = ',1p4e10.3,/,
     &        'z',/,
     &        't',/,
     &        'U',/,
     &        'V',/,
     &        'dL/dt',/,
     &        'L',/,
```

```
     &         'P',/,
     &         'Xtail')
    5 format (1p,8e10.3)
    6 format (' Screen Only or File Output too (S/F)? ')
    7 format (' Illegal Choice...Try again...')
    8 format (/,' Average number of series terms used in solution = ',i3
     &       ,/,' Number of times at which solution was evaluated = ',i3)
    9 format (' Requested timestep requires more than ',i5,
     &        ' solutions...',/,' Please increase MXPS PARAMETER')
   10 format (a1)
   11 format (' Solutions have been requested at more than',i4,' times.'
     & ,/,' MXPS parameter must be increased in order to proceed.')
   12 format (' Penetrator goes rigid, since  U = V.')
   13 format (' Rod continues to erode in rigid target, since V = 0.')
C
C     INPUT TATE CONSTANTS AND INITIALIZE CONSTANTS FOR CURRENT MODEL
      call getconstants
      rootgamma = dsqrt(gamma)
C
C     DETERMINE DESIRED DT STEP SIZE (NOT NEEDED TO SOLVE THE EQUATIONS;
C     RATHER, IT PROVIDES CONVENIENCE OF OUTPUT SPECIFICATION TO THE
C     USER)
      write (*,1)
      read (*,*) dt
C
C     INITIALIZE VARIABLES
      npass = 1
C
C     DETERMINE OUTPUT MODE (S FOR SCREEN ONLY; F FOR FILE OUTPUT TOO)
   70 write (*,6)
      read (*,10) answer
C     CONVERT ANSWER TO UPPER CASE
      if (answer .eq. 's') then
        answer = 'S'
      else if (answer .eq. 'f') then
        answer = 'F'
      end if
      if (answer .eq. 'F') then
C
C       SET UP OUTPUT FILE 'TATE.OUT'
        open (2,file='tate.out',access='append')
        write (2,4) R, Y, rhot, rhor
      else if (answer .ne. 'S') then
        write (*,7)
        goto 70
      end if
C
C     FOR FIRST ITERATION, START AT Z0
      z(0) = z0
C
C     AS LONG AS Z IS ABOVE MINIMUM LIMIT, PROCEED WITH THE FOLLOWING:
  100 if (z(npass-1) .gt. zx) then
C
C       COMPUTE DZ REQUIRED TO PRODUCE DESIRED T (APPROXIMATE ONLY)
C       BY EVALUATING DERIVATIVE AT MIDDLE OF Z INCREMENT
        zz = z(npass-1)
        do 105 i = 1, 2
          dz = dt
     &      * F / dexp(BigB*zz - c/zz)
     &      / (                        zz**((A-1.)/2.)
     &         + sigma*(1.-gamma)  *  zz**((A-3.)/2.))
        if (-dz .gt. zz) then
          zz = zz/2.
```

```
          else
            zz = zz + dz/2.
          end if
  105     continue
C
C         DETERMINE NEW Z.
C         IF Z FALLS BELOW MINIMUM LIMIT, RESET Z TO MINIMUM LIMIT
          z(npass) = z(npass-1) + dz
          if (z(npass) .lt. zx) z(npass) = zx
C
C         INCREMENT COUNTER ON NUMBER OF TIMES AT WHICH SOLUTION IS
C         COMPUTED
          npass = npass + 1
C
C         CHECK TO SEE THAT ARRAY LIMITS HAVE NOT BEEN EXCEEDED
          if (npass .gt. MXPS) then
            write (*,11) MXPS
            stop
          end if
          goto 100
        end if
C
C       EVALUATE T, FOR THE GIVEN Z, REQUIRING SOLUTION OF THE
C       W INTEGRAL
        D = (A-1.)/2.
        call wintegral (term1,npass,BigB,c,D,z0,z,nterms,EPSILON)
        ntavg = nterms
C
        D = (A-3.)/2.
        call wintegral (term2,npass,BigB,c,D,z0,z,nterms,EPSILON)
        ntavg = ntavg + nterms
C
        do 110 i = 0, (npass-1)
          t(i) = (term1(i) + sigma*(1.-gamma)*term2(i)) / F
  110   continue
C
C       EVALUATE PENETRATION, REQUIRING SOLUTION OF W INTEGRAL
        D = (A    )/2.
        call wintegral (term1,npass,BigB,c,D,z0,z,nterms,EPSILON)
        ntavg = ntavg + nterms
C
        D = (A-2.)/2.
        call wintegral (term2,npass,BigB,c,D,z0,z,nterms,EPSILON)
        ntavg = ntavg + nterms
C
        D = (A-4.)/2.
        call wintegral (term3,npass,BigB,c,D,z0,z,nterms,EPSILON)
        ntavg = (ntavg + nterms) / 5
C
        do 115 i = 0, (npass-1)
          P(i) = Uo / F * (
     &           (1-rootgamma)/(2.*rootgamma*(1.-gamma)) * term1(i) -
     &           sigma                                   * term2(i) -
     &           sigma**2*(1.+rootgamma)*(1.-gamma)/(2.*rootgamma)
     &                                                   * term3(i))
  115   continue
C
C       SET UP HEADER COLUMNS
        write (*,2)
C
C       DETERMINE REST OF PARAMETERS
        do 120 i = 0, (npass-1)
          zz = z(i)
```

```
C
C       EVALUATE U, FROM THE GIVEN Z
        Ur = (zz - sigma*(1.-gamma)) / (2.*rootgamma*dsqrt(zz))
        U = Ur * Uo
C
C       EVALUATE V FROM THE COMPUTED U
        Vr = (Ur - dsqrt(gamma*Ur**2 + sigma*(1.-gamma))) / (1.-gamma)
        V = Vr * Uo
C
C       EVALUATE Ldot FROM COMPUTED U AND V
        Ldot = V - U
C
C       EVALUATE ROD LENGTH FROM COMPUTED U
        rootterm = dsqrt(gamma*Ur**2 + sigma*(1.-gamma))
        rootz = Ur*rootgamma + rootterm
        L = Lo * dexp(Ur*(rootterm - Ur*gamma)/(2.*K*(1.-gamma)))
     &      * (rootz)**A / (dexp(N) * M)
C
C
C       COMPUTE TAIL LOCATION OF ROD
        xtail = P(i) - L
C
C       WRITE TO SCREEN
        write (*,3) z(i), t(i), U, V, Ldot, L, P(i), xtail
C       OUTPUT RESULTS
        if (answer .eq. 'F') then
C
C         WRITE TO FILE
          write (2,5) z(i), t(i), U, V, Ldot, L, P(i), xtail
        end if
  120 continue
C
C     COMPLETE THE PROBLEM TO ALLOW...
      i = npass
      z(i) = 0.
      U = 0.
      V = 0.
      Ldot = 0.
C
C     ...RIGID BODY ROD TO COME TO STOP, IF R < Y
      if (sigma .lt. 0.) then
        write (*,12)
        L = L
        P(i) = P(i-1) + L/gamma * dlog(Y/R)
        C1 = dsqrt( 2.*K*R/(gamma*Y) )
        t(i) = t(i-1) + 2.*L/(gamma*Uo*C1) *
     &         datan (Ur/C1)
C
C     ...OR NON-PENETRATING ERODING ROD TO COME TO STOP, IF R > Y
      else
        write (*,13)
        L = L * dexp ((Y-R)/Y)
        P(i) = P(i-1)
        z(0) = 0.
        BigB = 1./(2.*K)
        c = 0.
        E = 2.
        call wintegrate (term1,1,BigB,c,E,Ur,z,nterms,EPSILON)
        t(i) = t(i-1) - L/(2.*K*Uo) * term1(0)
      end if
      xtail = P(i) - L
C     WRITE TO SCREEN
      write (*,3) z(i), t(i), U, V, Ldot, L, P(i), xtail
```

```
C     OUTPUT RESULTS
      if (answer .eq. 'F') then
C
C       WRITE TO FILE
        write (2,5) z(i), t(i), U, V, Ldot, L, P(i), xtail
      end if
C
C     FINISHED ITERATING... CLOSE UP SHOP.
      write (*,8) ntavg, npass
      if (answer .eq. 'F') close (2)
      stop
      end
C***********************************************************************
      subroutine getconstants
C
C     ACQUIRE THE TATE CONSTANTS NEEDED BY THE CODE, AND INITIALIZE
C     CORRESPONDING CONSTANTS FOR PRESENT SOLUTION
C
C     -----VARIABLES-----
C
C     *****DIMENSIONAL TATE CONSTANTS*****
C
C     R    - TARGET RESISTANCE
C     Y    - ROD RESISTANCE
C     RHOT - TARGET DENSITY
C     RHOR - ROD DENSITY
C     Lo   - INITIAL ROD LENGTH
C     Uo   - INITIAL ROD VELOCITY
C
C     *****NON-DIMENSIONAL CONSTANTS*****
C
C     GAMMA
C     SIGMA
C     K
C     A
C     BIGB
C     C
C     M
C     N
C     Z0     - INITIAL VALUE OF Z (WHEN U = Uo)
C     ZX     - TERMINAL VALUE OF Z (WHEN V = 0)
C
C     *****OTHER VARIABLES*****
C
C       ...  - MINIMUM VALUE OF U, WHEN V = 0
C
      double precision gamma, sigma, K, A, BigB, c, F, z0, zx, M, N
      common /wconstants/ gamma, sigma, K, A, BigB, c, F, z0, zx, M, N
      double precision R, Y, rhot, rhor, Lo, Uo
      common /tconstants/ R, Y, rhot, rhor, Lo, Uo
      double precision Umin
C
    1 format (' Enter the target & rod resistances (R & Y): ')
    2 format (' Enter the target & rod densities: ')
    3 format (' Enter the initial rod length and velocity: ')
    4 format (1p,' The target resistance,',e10.3,
     &        ', must not equal the rod resistance.')
    5 format (1p,' The initial rod velocity,',e10.3,
     &        ', must exceed the minimum value,',e10.3,'.')
C
C     READ PARAMETERS REQUIRED IN TATE EQUATIONS
      write (*,1)
      read (*,*) R, Y
```

```
      write (*,2)
      read (*,*) rhot, rhor
      write (*,3)
      read (*,*) Lo, Uo
c
c     CHECK FOR DATA INCONSISTENCIES
c
c     TARGET RESISTANCE MUST NOT EQUAL ROD RESISTANCE, FOR
c     SOLUTION TO TATE EQUATIONS TO BE MEANINGFUL
      if (R .eq. Y) then
        write (*,4) R
        stop
      end if
c
c     Umin OCCURS WHEN PENETRATION VELOCITY V = 0 FOR R > Y, AND U = V
c     FOR R < Y.  IT ROUGHLY CORRESPONDS TO THAT VELOCITY WHERE THE ROD
c     BOUNCES OFF THE TARGET, WITHOUT MAKING PENETRATION.  MAKE SURE
c     THAT Uo EXCEEDS THE VALUE OF Umin.
      sigma = 2. * (R-Y) / (rhor*Uo**2)
      gamma = rhot / rhor
      if (sigma .gt. 0.) then
        Umin = Uo * dsqrt(sigma)
      else
        Umin = Uo * dsqrt(-sigma/gamma)
      end if
      if (Uo .lt. Umin) then
        write (*,5) Uo, Umin
        stop
      end if
c
c     INITIALIZE REST OF CONSTANTS (DERIVED FROM TATE CONSTANTS)
c
      call initialize
c
      return
      end
c***********************************************************************
      subroutine initialize
c
c     INITIALIZE CONSTANTS USED IN SOLUTION OF TATE PROBLEM,
c     GIVEN THE TATE CONSTANTS
c
c     =====VARIABLES=====
c
c     *****DIMENSIONAL TATE CONSTANTS*****
c
c     R    - TARGET RESISTANCE
c     Y    - ROD RESISTANCE
c     RHOT - TARGET DENSITY
c     RHOR - ROD DENSITY
c     Lo   - INITIAL ROD LENGTH
c     Uo   - INITIAL ROD VELOCITY
c
c     *****NON-DIMENSIONAL CONSTANTS*****
c
c     GAMMA
c     SIGMA
c     K
c     A
c     BIGB
c     C
c     M
c     N
```

```
c      Z0       - INITIAL VALUE OF Z (WHEN U = Uo)
c      ZX       - TERMINAL VALUE OF Z (WHEN V = 0)
c
c      *****OTHER VARIABLES*****
c
c      ROOTGAMMA - SQUARE ROOT OF GAMMA, WHICH OCCURS FREQUENTLY
c      ROOTTERM  - A SQUARE ROOT TERM, APPEARING OCCASIONALLY
c      ROOTZ0    - SQUARE ROOT OF Z0, APPEARING OCCASIONALLY
c      Udotor    - ORIGINAL ACCELERATION OF ROD / Uo
c
       double precision gamma, sigma, K, A, BigB, c, F, z0, zx, M, N
       common /wconstants/ gamma, sigma, K, A, BigB, c, F, z0, zx, M, N
       double precision R, Y, rhot, rhor, Lo, Uo
       common /tconstants/ R, Y, rhot, rhor, Lo, Uo
       double precision rootgamma, rootterm, rootz0, Udotor
c
c      GAMMA DEFINED IN ROUTINE GETCONSTANTS...NO NEED TO REINITIALIZE
c       gamma = rhot / rhor
c
c      TEMPORARY VARIABLE...
       rootgamma = dsqrt(gamma)
c      SIGMA DEFINED IN ROUTINE GETCONSTANTS...NO NEED TO REINITIALIZE
c      sigma = 2. * (R-Y) / (rhor*Uo**2)
       K     = Y / (rhor*Uo**2)
       A     = sigma / (2. * K * rootgamma)
       BigB  = 1. / (8. * K * rootgamma * (rootgamma + 1.))
       c     = sigma**2 * (1. - gamma) * (rootgamma + 1.) /
      &          (8. * K * rootgamma)
c
c      Z0 IS THE INITIAL VALUE OF Z, CORRESPONDING TO THE SITUATION
c      OF U = Uo.
       rootterm = dsqrt(gamma + sigma*(1.-gamma))
       rootz0 = rootgamma + rootterm
       z0     = rootz0**2
c
c      M AND N ARE CONSTANTS APPEARING IN THE EQUATION FOR dU/dt (AND THUS
c      THE EQUATION FOR U(t).
       M = (rootz0)**A
       N = (rootterm - gamma) / (2.*K*(1.-gamma))
c
       Udotor = -Y / (rhor * Lo * Uo)
       F = 4. * Udotor * M * rootgamma * dexp(N - sigma/(4. * K))
c
       if (R .gt. Y) then
c
c        FOR R > Y:
c        ZX IS THE TERMINAL VALUE OF Z, WHEN V = 0.  FROM TATE EQUATION,
c        WE SEE THAT V = 0 WHEN U/Uo = ROOT(SIGMA).  WHEN U IS THIS VALUE,
c        Z TAKES ON THE FOLLOWING VALUE:
         zx     = sigma * (rootgamma + 1.)**2
       else
c
c        FOR R < Y:
c        ZX IS THE TERMINAL VALUE OF Z, WHEN V = U.  FROM TATE EQUATION,
c        WE SEE THAT V = U WHEN U/Uo = ROOT(-SIGMA/GAMMA).  WHEN U IS
c        THIS VALUE, Z TAKES ON THE FOLLOWING VALUE:
         zx     = -sigma * (rootgamma + 1.)**2
       end if
c
       return
       end
c***********************************************************************
       subroutine wintegral (w, npass, B, C, D, z1, z2, n, eps)
```

```
C
C     SUBROUTINE TO EVALUATE THE W INTEGRAL:
C
C     z2
C     (    D            -1
C     |  z  exp (Bz - Cz   ) dz
C     )
C     z1
C
C     TO DO SO, TRANSFORM THE INTEGRAL WITH THE FOLLOWING TRANSFORMATION
C     AND INTEGRATE THAT FUNCTION, AS FOLLOWS:
C
C               D+1               1/(D+1)                     D
C     LET y = z    .  THEN z = y          AND dy = (D+1) z  dz.
C
C     LET E = 1/(D+1).  THE INTEGRAL THEN BECOMES
C
C       y2
C       (           E     -E
C     E |  exp (By - Cy    ) dy
C       )
C       y1
C
C     =====VARIABLES=====
C
C     *****BOOKKEEPING VARIABLES
C
C     MXPS  - MAXIMUM NUMBER OF TIMES, LESS 1, AT WHICH SOLUTION CAN
C              BE EVALUATED
C     EPS   - RELATIVE ERROR TO BE USED IN TESTING FOR SERIES
C              CONVERGENCE OF THE W INTEGRAL
C     N     - NUMBER OF TERMS REQUIRED TO CONVERGE THE W
C              INTEGRAL
C     I     - A LOOP INDEX
C     NPASS - NUMBER OF Z VALUES AT WHICH TO EVALUATE SOLUTION
C
C     *****OTHER VARIABLES*****
C
C     W()     - ARRAYS CONTAINING EVALUATIONS OF THE W INTEGRAL, EACH
C                ARRAY ELEMENT WITH DIFFERENT LIMITS OF INTEGRATION, AS
C                GIVEN BY THE Z2 ARRAY
C     Z1      - LOWER LIMIT OF INTEGRATION
C     Z2()    - ARRAY CONTAINING UPPER LIMITS OF INTEGRATION
C     B       - CONSTANT MULTIPLIER OF Z IN EXPONENTIAL TERM OF W
C               INTEGRAL
C     C       - CONSTANT MULTIPLIER OF 1/Z IN EXPONENTIAL TERM OF W
C               INTEGRAL
C     D       - EXPONENT OF Z IN W INTEGRAL
C     E       - TRANSFORMED VARIABLE EQUAL TO 1/(D+1)
C     Y1      - TRANSFORMED Z1 LIMIT OF INTEGRATION
C     Y2()    - TRANSFORMED Z2() ARRAY LIMITS OF INTEGRATION
C     DPLUS1  - INTERMEDIATE VARIABLE
C
      integer MXPS
      PARAMETER (MXPS=200)
      integer n, i, npass
      double precision w(0:MXPS), B, C, D, z1, z2(0:MXPS), eps
      double precision E, y1, y2(0:MXPS), Dplus1
      Dplus1 = D+1.
      y1 = z1**Dplus1
      do 100 i = 0, (npass-1)
  100   y2(i) = z2(i)**Dplus1
      E = 1./Dplus1
```

```
      call wintegrate (w, npass, B, C, E, y1, y2, n, eps)
      return
      end
C***********************************************************************
      subroutine wintegrate (w, npass, B, C, E, y1, y2, n, eps)
C
C     FUNCTION TO EVALUATE THE W INTEGRAL (WITH
C     TRANSFORMATION), BY SERIES EXPANSION:
C
C       y2
C       (             E     -E
C     E |  exp  (By  - Cy   ) dy
C       )
C       y1
C
C     THE SERIES EXPANSION OF THE EXPONENTIAL IS:
C
C                               2        3
C     exp  (x) = 1 + x + x /2! + x /3! ...
C
C                  ^    ^    ^        ^
C                  |    |    |        |
C     TERM #       0    1    2        3
C
C     WHEN THE ABSOLUTE VALUE OF A GIVEN TERM OF THIS EXPANSION IS
C     LESS THAN THE ERROR "eps" TIMES THE CURRENT FUNCTION VALUE,
C     THE SERIES IS TRUNCATED.  THE NUMBER OF TERMS REQUIRED TO
C     SUM WITHIN THIS ERROR VALUE IS RETURNED IN VARIABLE "n".
C
C     NOTE THAT INTEGRAL OF TERM 0 OF THE EXPANSION IS (y2-y1) WHEN
C     EVALUATED BETWEEN y1 AND y2.
C
C     REMEMBER ALSO, THAT TERM "x" IN THIS EXPANSION IS REALLY A TERM
C     LIKE (A + B). THUS, THE POLYNOMIAL EXPANSION OF TERMS IN THE
C     EXPONENTIAL ARE:
C
C              n     n      n     k (n-k)
C     (A + B)   = SUM   (     ) A B
C                 k=0     k
C
C     WHERE:
C
C       n          n!
C     (   ) = -----------
C       k      (n-k)! k!
C
      integer MXPS
      PARAMETER (MXPS=200)
      integer n, k, npass, i
      double precision w(0:MXPS), B, C, E, y1, y2(0:MXPS), eps
      double precision f, fi, exponent, exp2,
     &                  value(0:MXPS), tvalue(0:MXPS), minlimit
C
C     W INTEGRAL ONLY INTENDED FOR POSITIVE LIMIT OF INTEGRATION
      minlimit = y1
      do 80 i = 0, (npass-1)
   80   if (y2(i) .lt. minlimit) minlimit = y2(i)
C     CHECK TO SEE IF INTEGRATION LIMITS ARE OK
      if (minlimit .lt. 0.) then
        write (*,'('' Negative integration limits not allowed.'')')
        stop
      end if
C
```

```
C     INITIALIZE COUNTER FOR # OF TERMS OF EXPONENTIAL EXPANSION USED
      n = 0
C
C     EVALUATE THE INTEGRAL OF THE FIRST TERM DIRECTLY:
      do 90 i = 0, (npass-1)
   90   w(i) = (y2(i) - y1)
C
C       FOR EVERY ADDITIONAL TERM OF THE EXPONENTIAL EXPANSION, DO THE
C       FOLLOWING:
C       INCREMENT TERM COUNTER
  100   n = n + 1
C
C       INITIALIZE INTEGRAL ASSOCIATED WITH THIS EXPONENTIAL TERM TO ZERO
        do 110 i = 0, (npass-1)
  110     tvalue(i) = 0.
C
C       LOOP OVER EACH POLYNOMIAL EXPANSION TERM, FOR THIS EXPONENTIAL
C       TERM
        do 200 k = 0, n
C
C         GET THE CONSTANT ASSOCIATED W/ THIS EXPONENTIAL/POLYNOMIAL
C         PRODUCT TERM...IT IS GIVEN BY:
C         f = B^k * (-C)^(n-k) / (n-k)! k!
C         THE FOLLOWING TECHNIQUE REDUCES PROBABILITY OF NUMERICAL
C         OVERFLOW, BY INTERSPERSING DIVISION AND MULTIPLICATION
          f = 1.
          do 120 i = 1, max0(k,(n-k))
            fi = dfloat(i)
            if (i .le. k  ) f = f *   b /fi
            if (i .le. n-k) f = f * (-c)/fi
  120     continue
          if (f .ne. 0.) then
C
C           GET THE EXPONENT ON Y ASSOCIATED W/ THIS POLYNOMIAL EXPANSION
C           TERM
            exponent = E * (2*k - n)
C
C           BEGIN THE INTEGRATION OF THIS POLYNOMIAL EXPANSION TERM:
C
C           y2
C           (       n  k  (n-k)  E(2k-n)             y2
C           |   (-1)  B  C        y                  (      exponent
C           |  ------------------------- dy   =     |  f y          dy
C           )         (n-k)! k!                      )
C           y1                                       y1
C
            if (exponent .ne. -1.) then
C             IF EXPONENT NOT EQUAL TO -1, THEN
C
C             y2
C             (      exponent              f              (exponent+1)  |y2
C             |  f y          dy = --------------     y                 |
C             )                    (exponent + 1)                       |y1
C             y1
C
C             FOR NORMAL POLYNOMIAL INTEGRATION...
C             GET THE EXPONENT AFTER INTEGRATION
              exp2 = exponent + 1.
C             GET THE TOTAL COMBINED CONSTANT AFTER INTEGRATION
              f = f / exp2
              do 150 i = 0, (npass-1)
C               EVALUATE THE INTEGRAL BETWEEN Y1 AND Y2
                value(i) = f * (y2(i)**exp2 - y1**exp2)
```

```
c              LUMP THIS INTEGRATION VALUE FOR THIS POLYNOMIAL TERM INTO
c              THE SUM FOR THE EXPONENTIAL TERM
               tvalue(i) = tvalue(i) + value(i)
  150        continue
           else
c            OTHERWISE, IF EXPONENT = -1, THEN
c
c            y2
c            (      exponent
c            |  f y          dy = f ln(y2/y1)
c            )
c            y1
c
c            FOR LOGARITHMIC INTEGRATION...
c            THE TOTAL COMBINED CONSTANT AFTER INTEGRATION IS JUST F
             do 160 i = 0, (npass-1)
c              EVALUATE THE INTEGRAL BETWEEN Y1 AND Y2
               value(i) = f * dlog(y2(i)/y1)
c              LUMP THIS INTEGRATION VALUE FOR THIS POLYNOMIAL TERM INTO
c              THE SUM FOR THE EXPONENTIAL TERM
               tvalue(i) = tvalue(i) + value(i)
  160        continue
           end if
         end if
c
  200   continue
c
c       LUMP THIS INTEGRATION VALUE FOR THE EXPONENTIAL TERM INTO
c       THE SUM FOR THE W INTEGRAL
        do 250 i = 0, (npass-1)
  250     w(i) = w(i) + tvalue(i)
c
c       CHECK FOR SERIES CONVERGENCE AGAINST USER SPECIFIED EPSILON;
c       IF TERM DOESN'T CONVERGE FOR ANY OF THE LIMITS OF INTEGRATION,
c       THEN COMPUTE ANOTHER EXPONENTIAL SERIES TERM FOR *ALL* THE
c       LIMITS OF INTEGRATION
        do 300 i = 0, (npass-1)
  300     if (dabs(tvalue(i)) .gt. dabs(eps*w(i))) goto 100
c
c       MODIFY INTEGRAL VALUE TO ACCOUNT FOR E CONSTANT IN FRONT
c       OF INTEGRAL
        do 350 i = 0, (npass-1)
  350     w(i) = E * w(i)
      return
      end
c*****************************************************************************
```

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